CHAPTER 03 · MULTI-QUBIT CIRCUITS

CNOT, Bell states, your *first* quantum program.

Topic 02 gave you single-qubit gates on the Bloch sphere. Now we take the step that makes quantum computing actually quantum — two qubits, entangled, on real hardware.

WHAT YOU WILL LEARN

- Why CNOT and CZ entangle
- Build a Bell state in Qiskit
- What the transpiler does

— 02 · TENSOR PRODUCT

Two qubits live in $C^2 \otimes C^2$ — four basis states.

$|00\rangle$

BASIS 0

$|01\rangle$

BASIS 1

$|10\rangle$

BASIS 2

$|11\rangle$

BASIS 3

Any two-qubit pure state

$$|\Psi\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

Four complex amplitudes — a, b, c, d — with $|a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$. With N qubits you get 2^N basis states. That exponential headroom is what makes classical simulation hard fast.

- ● 1 qubit · dim 2
- ● 2 qubits · dim 4
- ● 50 qubits · dim $\sim 10^{15}$

03 · ENTANGLING GATES

CNOT and CZ — the two workhorses.

CNOT · CONTROLLED-NOT

Flip target iff control is 1

```
00 → 00
01 → 01
10 → 11
11 → 10
```

Native on trapped ions via Mølmer-Sørensen. Compiled from CZ on superconducting.

CZ · CONTROLLED-Z

Phase flip iff both are 1

```
00 → +00
01 → +01
10 → +10
11 → -11
```

Native on superconducting transmons — Google Willow, IBM Heron, Zuchongzhi.

Applied to a superposition, either gate takes a separable state to one that cannot be factored back. That is entanglement.

— 04 · BELL BASIS

Four maximally entangled two-qubit states.

Φ^+

PHI PLUS

$(|00\rangle + |11\rangle) / \sqrt{2}$

$H_0 \cdot CX_{0,1}$ on $|00\rangle$

Φ^-

PHI MINUS

$(|00\rangle - |11\rangle) / \sqrt{2}$

X_0 first, then Φ^+ recipe

Ψ^+

PSI PLUS

$(|01\rangle + |10\rangle) / \sqrt{2}$

X_1 first, then Φ^+ recipe

Ψ^-

PSI MINUS

$(|01\rangle - |10\rangle) / \sqrt{2}$

$X_0 + X_1$ first, then Φ^+

Orthonormal basis for two-qubit entanglement. Every teleportation, superdense-coding, and entanglement-swapping protocol uses one of these four.

05 · CODE ANCHOR · QISKIT 1.X

Five lines. A real Bell state.

BELL.PY

```python
from qiskit import QuantumCircuit
from qiskit_aer.primitives import SamplerV2

qc = QuantumCircuit(2, 2)
qc.h(0)
qc.cx(0, 1)
qc.measure([0, 1], [0, 1])

result = SamplerV2().run([qc], shots=1024).result()
print(result[0].data.c.get_counts())
```

Histogram out

~50%
$|00\rangle$

~50%
$|11\rangle$

0%
$|01\rangle$

0%
$|10\rangle$

Hadamard puts qubit 0 in equal superposition. CNOT correlates qubit 1. Measure either one, the other matches — every time.

DIALOGUE · NO-SIGNALLING, NO-CLONING

Does entanglement let me signal faster than light?

Ava · host

live debate

If measuring one Bell pair instantly fixes the other — does that not let me send a signal faster than light?

The two theorems that close the loophole

Why relativity and quantum mechanics stay friends

↓

↓

3 **Consistent with relativity**
Correlation, not communication — verified in every Bell test 1972 → 2022

07 · TRANSPILATION

Same circuit. Different native gates. Per backend.

YOUR CIRCUIT

```
qc = QuantumCircuit(2)
qc.h(0)
qc.cx(0, 1)
# 2 logical gates
```

Clean. Conceptual. Not what the hardware actually runs.

TRANSPILE(QC, BACKEND=IBM_HERON)

```
RZ(π/2) · SX · RZ(π/2)   # H decomposed
RZ · SX · CZ · SX · RZ   # CX decomposed
# native set: CZ, RZ, SX, X
```

Heron R2 has no H, no CX. Two logical gates become six or seven physical ones — and every extra gate is more time for decoherence.

The transpiler is also the optimizer. It decides whether your circuit fits inside the qubit's coherence budget.

08 · CIRCUIT COMPLEXITY

Depth costs more than width on noisy hardware.

width

QUBITS IN PARALLEL

How many qubits the circuit acts on at once. Cheap to scale on neutral-atom and ion arrays.

depth

SEQUENTIAL LAYERS

How many gate layers in series. Every layer is more time, more decoherence, more two-qubit error.

cost

DEPTH × 2Q-ERROR

A 100-wide depth-1000 circuit is far harder than a 10-wide depth-100 — same gate count, very different fidelity.

Practical rule for 2026 hardware — when you can pick, parallelize over qubits rather than stack rounds.

— 09 · HARDWARE CAPABILITY · 2024 → 2026

Mid-circuit measurement — what 2026 silicon enables.

Measurement during a circuit, not just at the end

Three milestones that made it production

↓

↓

3 **IBM Heron R2 — Qiskit Runtime, 2025**
Mid-circuit measurement exposed to general users; enables dynamic circuits and feedforward control

Real-time quantum error correction is unlocked by this one feature. It is the gate-level difference between 2023 hardware and 2026 hardware.

— 10 · TRY IT YOURSELF

Build the GHZ state on 3 qubits.

— GHZ.PY

```
qc = QuantumCircuit(3, 3)
qc.h(0)
qc.cx(0, 1)
qc.cx(1, 2)
qc.measure([0,1,2], [0,1,2])
```

Three-party generalisation of the Bell state — the Greenberger-Horne-Zeilinger state.

Predict the histogram

~50%

$|000\rangle$

~50%

$|111\rangle$

- **Aer simulator · zero shots cost**
- **IBM Quantum free tier · 10 min/month on Heron**

Next up — Chapter 04. Superconducting silicon. Willow, Heron, Zuchongzhi — what they are made of and what their limits actually are.