CHAPTER 02 · FOUNDATIONS

Qubits & single-qubit gates — the *linear algebra* that actually matters.

By the end of these ten slides you will have written your first quantum operation in Qiskit — and you will know exactly what superposition is, what it does, and what it is not.

WHAT YOU WILL LEARN

02 · THE QUBIT, FORMALLY

A unit vector in two complex dimensions.

DEFINITION

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

$\alpha, \beta \in \mathbb{C}$ with $|\alpha|^2 + |\beta|^2 = 1$ — the normalisation constraint.

Two complex amplitudes, one real constraint, one global-phase irrelevance — net two real degrees of freedom.

WORKED EXAMPLE

$$\alpha = \sqrt{3} / 2 \;\cdot\; \beta = 1 / 2$$

P(measure 0) = $|\alpha|^2$ = **0.75**

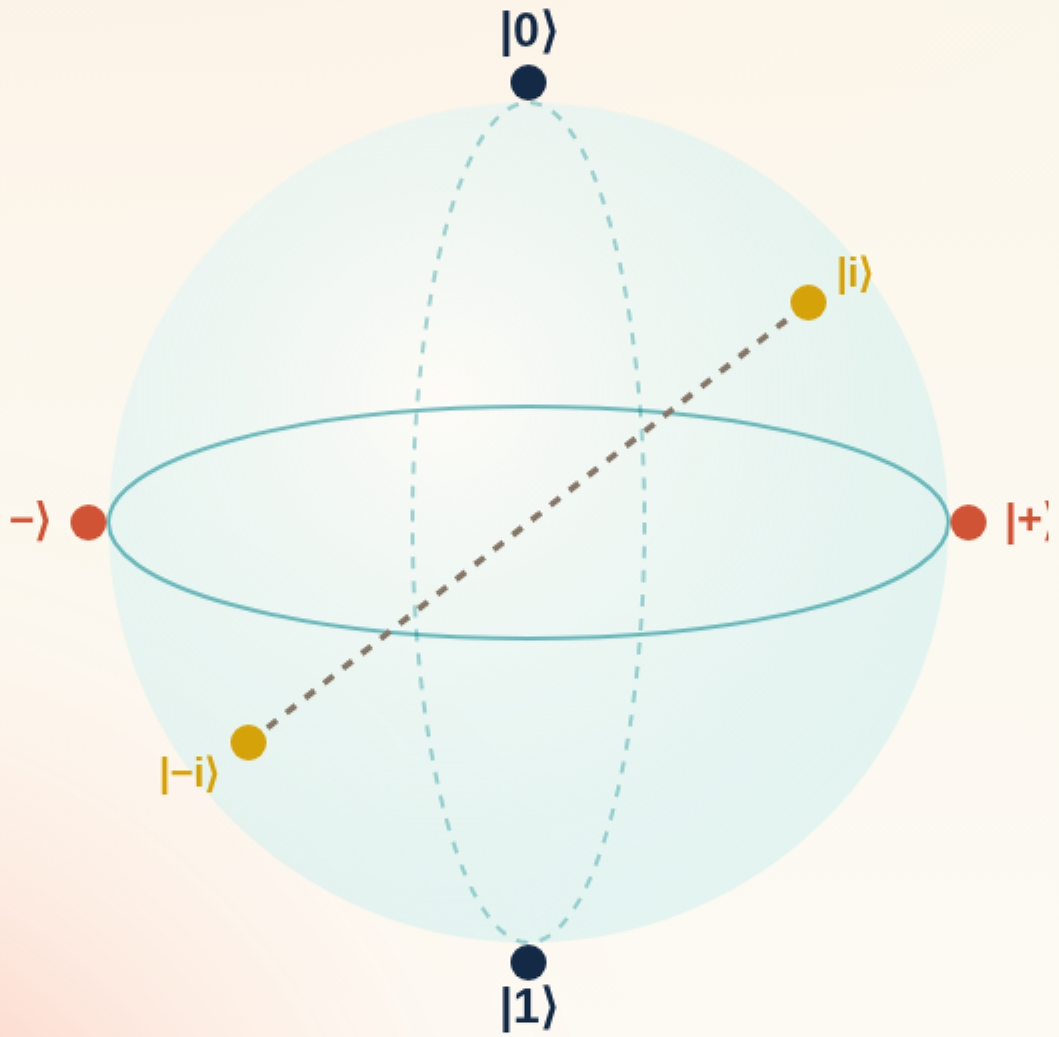
P(measure 1) = $|\beta|^2$ = **0.25**

Amplitudes square to probabilities — the Born rule, in two lines.

Why complex numbers and not just a probability vector? Because amplitudes can be negative and can carry phase — and they interfere. Probabilities only add.

03 · GEOMETRIC PICTURE

Every pure single-qubit state lives on a sphere.



READING THE SPHERE

Angle θ from the z-axis sets the measurement probabilities. Angle φ around the equator sets the relative phase between |0⟩ and |1⟩.

$|\psi\rangle = \cos(\theta/2)\,|0\rangle + e^{i\varphi}\sin(\theta/2)\,|1\rangle$

- **Poles · |0⟩ & |1⟩**
- **x-axis · |+⟩ & |−⟩**
- **y-axis · |i⟩ & |−i⟩**

One qubit, infinite states on the surface — but only one classical bit ever comes out at measurement.

04 · THE SINGLE-QUBIT ALPHABET

Pauli X, Y, Z — three matrices, every other unitary follows.

X

BIT-FLIP · QUANTUM N O T

$X|0\rangle = |1\rangle$ · $X|1\rangle = |0\rangle$

```
[ 0  1 ]
[ 1  0 ]
```

Y

BIT-FLIP & PHASE-FLIP

$Y = i \cdot X \cdot Z$ · $Y|0\rangle = i|1\rangle$

```
[ 0  −i ]
[ i   0 ]
```

Z

PHASE-FLIP

$Z|0\rangle = |0\rangle$ · $Z|1\rangle = -|1\rangle$

```
[ 1   0 ]
[ 0  −1 ]
```

All three are Hermitian, unitary, and self-inverse. Every single-qubit unitary decomposes into a product of Paulis and rotation angles — these three are the alphabet, the rest is grammar.

05 · THE SUPERPOSITION GATE

Hadamard — the single most important gate in this course.

MATRIX

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

H is Hermitian, unitary, and its own inverse — H · H = I. Apply it twice and you are back where you started.

WORKED EXAMPLE

$H\,|0\rangle = (|0\rangle + |1\rangle)\,/\,\sqrt{2} = |+\rangle$

$H\,|1\rangle = (|0\rangle - |1\rangle)\,/\,\sqrt{2} = |-\rangle$

Both outputs measure 50/50 between 0 and 1 — but they differ by a sign on $|1\rangle$. That sign is the phase that lets quantum algorithms interfere.

H is the gate at the heart of the parallelism story — and the parallelism story is the most-misunderstood part of the field. Next slide: we write it in real code and watch what comes out.

06 · FIRST QUBIT · QISKIT 1.X

Six lines — Hadamard, measure, 1024 shots.

QISKIT_FIRST_QUBIT.PY

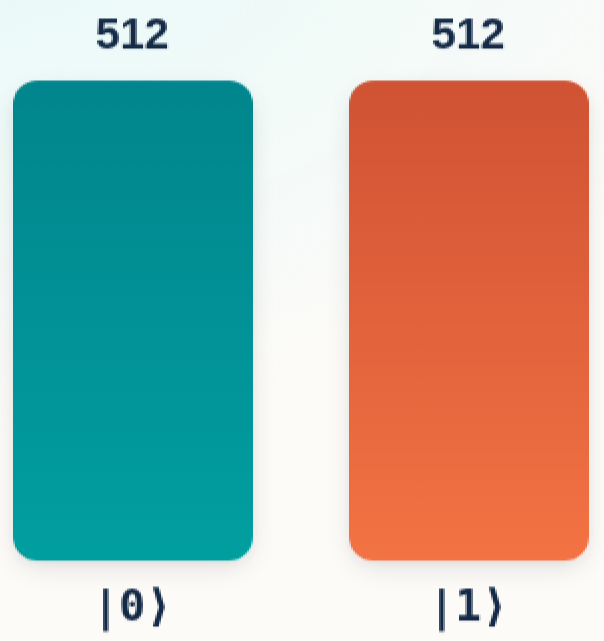
```
from qiskit import QuantumCircuit
from qiskit_aer.primitives import SamplerV2 as Sampler

qc = QuantumCircuit(1)
qc.h(0)
qc.measure_all()

result = Sampler().run([qc], shots=1024).result()
counts = result[0].data.meas.get_counts()
print(counts)   # {'0': ~512, '1': ~512}
```

Tested on Qiskit 1.3 with qiskit-aer 0.15. The exact split varies per run — sampling noise around a true 50/50.

MEASURED HISTOGRAM



Why 1024 shots and not one? One measurement gives one bit. The probability distribution only emerges from the ensemble.

07 · MEASUREMENT

The Born rule, collapse, and why one shot tells you nothing.

THE BORN RULE

P(outcome k) = $|\langle k \mid \psi \rangle|^2$

If $\alpha = \sqrt{3}/2$, then $P(0) = 3/4$. The squared magnitude of the amplitude. Two lines of arithmetic, one century of philosophy.

COLLAPSE

After measurement the state becomes whichever outcome occurred — the superposition is gone, and the unitary evolution that built it cannot be reversed from the measured state.

- Irreversible
- One-shot resource

THE LINE

This is the moment where quantum mechanics stops being linear algebra and starts being physics. Superposition is fragile — you build it, you use it, you measure once. Every algorithm in this course is shaped by that single fact.

DIALOGUE · AMPLITUDES VS PROBABILITIES

Is a qubit just a spinning coin?

Ava · host

amplitude check

A qubit in superposition is just a coin spinning in the air — half heads, half tails. Same thing, fancier vocabulary, right?

Two coins vs two qubits

Why the analogy breaks at "combine"

↓

↓

Square at the very end
Probabilities only appear at measurement — never before

09 · THE MISCONCEPTION THAT HURTS THE FIELD

Quantum is not parallel — it is *interferential*.

THE MYTH

A qubit in superposition computes every branch in parallel and hands you all the answers at once.

Wrong on both ends. You apply a unitary, the state evolves, you measure — and measurement collapses to one classical outcome.

WHAT REAL ALGORITHMS ACTUALLY DO

Arrange the interference so the **correct answer has high amplitude** and the wrong answers cancel. Then a single measurement returns the right answer with high probability.

Aaronson has been making this point in print for twenty years. The slogan: it is not parallelism, it is interference. The word matters — it is the difference between a marketing slide and a working algorithm.

- One unitary at a time
- One measurement, one bitstring
- Interference does the work

10 · TRY IT YOURSELF

Build a 75/25 measurement split — your first non-uniform circuit.

EXERCISE · RUN BEFORE CHAPTER 3

Write a one-qubit circuit that gives P(0) = 0.75 and P(1) = 0.25 when measured. Not fifty-fifty — biased on purpose.

```
from qiskit import QuantumCircuit
from math import pi

qc = QuantumCircuit(1)
qc.ry(pi/3, 0)        # <-- the gate you need
qc.measure_all()
# Submit to Aer first, then IBM Quantum (open plan)
```

Why pi/3? Because $\cos(\pi/6) = \sqrt{3}/2$, so amplitude of $|0\rangle$ squared equals 3/4. Geometry on the Bloch sphere becomes statistics at the histogram.

WHERE TO RUN IT

Run on the free Aer simulator first to verify the split. Then submit to real I B M Quantum hardware through the open plan — and compare the two histograms side by side.

- qiskit-aer (local, free)
- IBM Quantum · Open plan

Next chapter — two qubits together, the C N O T gate, and the Bell state. The first genuinely non-classical thing this course will do.